JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 25, No. 4, July–August 2002

# Output Feedback Form and Adaptive Stabilization of a Nonlinear Aeroelastic System

Sahjendra N. Singh* and Ling Wang†
*University of Nevada, Las Vegas, Las Vegas, Nevada 89154-4026*

**The question of output feedback representation and the design of a new adaptive control system for the control of an aeroelastic system using a single output feedback are treated. The chosen dynamic model describes the nonlinear plunge and pitch motion of a wing. The parameters of the system are assumed to be completely unknown. For the derivation of control law, the existence of output feedback forms of the model is examined. It is shown that for the choice of pitch angle as an output, an output feedback form of the system can be derived, but this kind of representation is not possible if the plunge displacement is chosen as an output. As such, adaptive control of the aerolastic model based on the backstepping design technique by plunge displacement feedback is not feasible. Then a global diffeomorphism is constructed for obtaining an output feedback form of the model when the pitch angle is the output. Based on this output feedback form and a backstepping design technique, an adaptive control law for the trajectory control of the pitch angle is derived. For the synthesis of the controller, only the pitch angle is used. It is shown that, in the closed-loop system, pitch angle trajectory control is accomplished and that the state vector asymptotically converges to the origin in spite of the uncertainties in the model using only pitch angle feedback.**

## Nomenclature

| | | |
|---|---|---|
| $A$, $A_0$, $\boldsymbol{l}$ | = | system matrices |
| $a$ | = | nondimensionalized distance from the midchord to the elastic axis |
| $b$ | = | semichord of the wing |
| $c_h$ | = | structural damping coefficient in plunge due to viscous damping |
| $c_\alpha$ | = | structural damping coefficient in pitch due to viscous damping |
| $h$ | = | plunge displacement |
| $I_\alpha$ | = | mass moment of inertia of the wing about the elastic axis |
| $k_h$ | = | structural spring constant in plunge |
| $k_\alpha$ | = | structural spring constant in pitch |
| $m_T$ | = | total mass of the wing and its support structure |
| $m_W$ | = | mass of the wing |
| $\boldsymbol{q}$, $\boldsymbol{x}$, $\tilde{\boldsymbol{x}}$ | = | state variables, estimation error |
| $U$ | = | freestream velocity |
| $V_i$ | = | Lyapunov functions |
| $\boldsymbol{v}_i$, $\boldsymbol{s}_i$ | = | components of $\boldsymbol{\Omega}$ |
| $x_\alpha$ | = | nondimensionalized distance measured from the elastic axis to the center of mass |
| $\alpha$ | = | pitch angle |
| $\beta$ | = | flap deflection |
| $\Gamma_i$, $c_i$, $\hat{l}_i$, $d_i$ | = | design parameters |
| $\boldsymbol{\theta}$, $\rho$; $\hat{\boldsymbol{\theta}}$, $\hat{\rho}$ | = | parameters; estimate of parameters |
| $\boldsymbol{\xi}$, $\boldsymbol{\Omega}$ | = | filter states |
| $\rho$ | = | density of air |

## I. Introduction

AEROELASTIC systems exhibit a variety of phenomena including instability, limit cycle, and even chaotic vibration.[1–3] Active control of aeroelastic instability is an important problem. Researchers have analyzed the stability properties of aeroelastic systems and designed controllers for flutter suppression. Digital adaptive control of a linear autoregressive moving average aeroservoelastic model has been considered.[4] At NASA Langley Research Center, a benchmark active control technique (BACT) wind-tunnel model has been designed, and control algorithms for flutter suppression have been developed.[5–10] In Refs. 6 and 7, unsteady aerodynamic data and flutter instability for the BACT project model are described. A robust flutter-suppression control law using the classical and minmax methods has been derived.[8] Robust passification techniques have been used in Ref. 9 for control. Two linear parameter-varying, gain scheduled controllers have been designed in Ref. 10. A computational two-dimensional aeroservoelasticity study has been done.[11] Active control of transonic wind-tunnel model using neural networks has been considered.[12] For an aeroelastic apparatus, control systems have been designed using feedback linearization and adaptive control strategies.[13–18] Adaptive control of an aeroelastic system using both plunge displacement and pitch angle feedback has been considered.[16] An output feedback adaptive controller using a high-gain observer has been designed.[18] The Euler–Lagrange theory for controlling an aeroelastic model has been used.[19]

A question of interest arises: Is it possible to design adaptive control systems for the stabilization of the aeroelastic system using a single measurement? If this can be done, then stabilization will be accomplished by utilizing a single sensor. Because pitch angle and plunge displacement are the two key variables that describe the dynamics of the aeroelastic model, one would like to explore if the pitch angle or the plunge displacement can be used for feedback control synthesis and stabilization.

The contribution of this paper lies in the derivation of an output feedback form and the design of an adaptive control law for the control of a nonlinear aeroelastic system using a single output measurement. A single trailing-edge control surface is used for the control of the system. For the output feedback design, it is essential to obtain an output feedback form of the aeroelastic model for an appropriate choice of an output variable. It is shown that, for the model with plunge displacement as an output, its output feedback form does not exist, and based on the backstepping design technique,[20,21] a feedback linearizing adaptive law using plunge displacement feedback is not feasible. Then for the choice of the pitch angle as an output, a global diffeomorphism for transforming the model into an output feedback form is derived. Based on a backstepping design technique[21] and the output feedback form of the model, an adaptive control law for the control of the pitch angle trajectory is developed. It is shown that, in the closed-loop system, the state vector of the aeroelastic system asymptotically converges to the origin in spite of the uncertainties in the system using only pitch angle measurement.

Received 14 February 2001; revision received 30 December 2001; accepted for publication 8 January 2002. 

*Professor, Electrical and Computer Engineering Department. Associate Fellow AIAA.
†Graduate Student, Electrical and Computer Engineering Department.

Interestingly, Mukhopadhyay[8] has shown that although pitch and pitch rate feedback can suppress the flutter of a NACA 0012 wing model, for robustness one also needs to feedback the plunge acceleration. Here adaptive control system uses only pitch angle for synthesis, but filters are designed that estimate all of the state variables of a related system for feedback. The filter states include the effect of plunge motion. The pitch angle can be derived from signals measured by two accelerometers.[8] Note that this adaptive design does not require any knowledge of system parameters nor does it impose any restriction on the bounds of the uncertain parameters. Thus, the aerodynamic force and moment and the damping and spring constants in plunge and pitch, as well as the mass and inertia of the wing, are assumed to be completely unknown to the designer. However, it is assumed that uncertain functions are linearly parameterized. Also note that this adaptive pitch angle feedback design assumes that only the unknown plunge spring is linear because it is not possible to obtain an output feedback form of the model for design if it has plunge spring nonlinearity.

## II. Aeroelastic Model and Control Problem

The prototypical aeroelastic wing section is shown in Fig. 1. The governing equations of motion are provided in Ref. 17 and are given by

$$\begin{bmatrix} m_T & m_W x_\alpha b_s \\ m_W x_\alpha b_s & I_\alpha \end{bmatrix}\begin{bmatrix} \ddot{h} \\ \ddot{\alpha} \end{bmatrix} + \begin{bmatrix} c_h & 0 \\ 0 & c_\alpha \end{bmatrix}\begin{bmatrix} \dot{h} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} k_h & 0 \\ 0 & k_\alpha(\alpha) \end{bmatrix}\begin{bmatrix} h \\ \alpha \end{bmatrix} = \begin{bmatrix} -L \\ M \end{bmatrix} \tag{1}$$

In Eq. (1), $M$ and $L$ are the aerodynamic lift and moment. It is assumed that the quasi-steady aerodynamic force and moment are of the form

$$L = \rho U^2 b_s c_{l_\alpha}\left[\alpha + (\dot{h}/U) + \left(\tfrac{1}{2} - a\right) b_s(\dot{\alpha}/U)\right] + \rho U^2 b_s c_{l_\beta}\beta$$

$$M = \rho U^2 b_s^2 c_{m_\alpha}\left[\alpha + (\dot{h}/U) + \left(\tfrac{1}{2} - a\right) b_s(\dot{\alpha}/U)\right] + \rho U^2 b_s^2 c_{m_\beta}\beta \tag{2}$$

where $c_{l_\alpha}$ and $c_{m_\alpha}$ are the lift and moment coefficients per angle of attack and $c_{l_\beta}$ and $c_{m_\beta}$ are lift and moment coefficients per control surface deflection. Although, other forms of nonlinear spring stiffness associated with the pitch motion can be considered, for purposes of illustration, the function $k_\alpha(\alpha)$ is considered as a polynomial nonlinearity given by

$$k_\alpha(\alpha) = 6.833 + 9.967\alpha + 667.685\alpha^2 + 26.569\alpha^3 - 5087.931\alpha^4$$

Defining the state vector $\boldsymbol{q} = (\alpha, h, \dot{\alpha}, \dot{h})^T$, one obtains a state variable representation of Eq. (1) in the form

$$\dot{\boldsymbol{q}} = \begin{bmatrix} 0_{2\times 2} & I_{2\times 2} \\ M_1 & M_2 \end{bmatrix}\boldsymbol{q} + \begin{bmatrix} 0_{2\times 1} \\ \boldsymbol{p}_0 \end{bmatrix} k_{n_\alpha}(\alpha) + \begin{bmatrix} 0_{2\times 1} \\ \boldsymbol{g}_0 \end{bmatrix}\beta$$

$$\doteq Q\boldsymbol{q} + \bar{\boldsymbol{p}} k_{n_\alpha}(\alpha) + \bar{\boldsymbol{g}}\beta \tag{3}$$

where $\alpha k_\alpha = \alpha k_{\alpha 0} + k_{n_\alpha}$, $k_{n_\alpha} = k_{\alpha_1}\alpha^2 + k_{\alpha_2}\alpha^3 + k_{\alpha_3}\alpha^4 + k_{\alpha_4}\alpha^5$, $\boldsymbol{p}_0 = (p_{01}, p_{02})^T$, $\boldsymbol{g}_0 = (g_{01}, g_{02})^T$, $\bar{\boldsymbol{g}} = (0^T, \boldsymbol{g}_0^T)^T$, $\bar{\boldsymbol{p}} = (0^T, \boldsymbol{p}_0^T)^T$, 0 and $I$ denote null and identity matrices of appropriate dimensions and

$$M_1 = \begin{bmatrix} -\left(k_4 U^2 + m_T d^{-1} k_{\alpha 0}\right) & -k_3 \\ -\left(k_2 U^2 - m_W x_\alpha b_s d^{-1} k_{\alpha 0}\right) & -k_1 \end{bmatrix}$$

$$M_2 = \begin{bmatrix} -c_{41} & -c_{31} \\ -c_{21} & -c_{11} \end{bmatrix}$$

The expression for the parameters $k_i$, $d$, $c_{ij}$, $g_{0k}$, and $f_{0k}$ are given in the Appendix. It is assumed that all of the matrices and parameters $M_i$, $\boldsymbol{p}_0$, $\boldsymbol{g}_0$, and $k_{\alpha i}$, $i = 0, 1, \ldots, 4$, of the model equation (3) are unknown.

Let

$$f(\boldsymbol{q}) = Q\boldsymbol{q} + \bar{\boldsymbol{p}} k_{n_\alpha}(\alpha) \tag{4}$$

Writing Eq. (3) in a compact form yields

$$\dot{\boldsymbol{q}} = \boldsymbol{f}(\boldsymbol{q}) + \bar{\boldsymbol{g}}\beta \tag{5}$$

Associated with system equation (5), consider an output

$$y = c(\boldsymbol{q}), \qquad y \in R \tag{6}$$

and a smooth reference trajectory $y_r$ converging to zero. The output $y$ is chosen such that trajectory tracking of $y_r$ accomplishes stabilization of the aerolastic system, that is, as the tracking error $y - y_r$ tends to zero, $\boldsymbol{q}(t)$ also converges to zero. Here the objective is to derive an adaptive control law for the trajectory control of $y$ that requires a minimal set of measurements (possibly a single measurement $y = h$ or $y = \alpha$) for the synthesis of the controller.

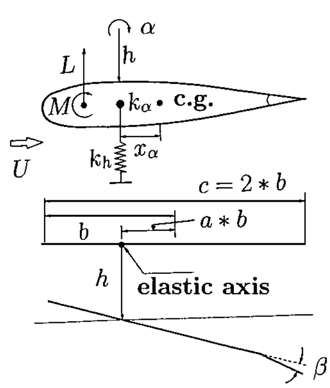



**Fig. 1 Aeroelastic model.**

## III. Output Feedback Form, Filters, and State Estimation

For output feedback linearizing adaptive control of the system, it is essential to obtain an output feedback form of the model by a suitable change of coordinates. However, not all systems can be transformed into output feedback forms. Here we explore the possibility of transforming Eq. (5) into an appropriate output feedback form by the choice of the plunge displacement or pitch angle as an output variable.

### A. Existence of Output Feedback Form

This subsection examines the question of existence of an output feedback form for the model. We seek a diffeomorphism $T$: $U_0 \to R^4$,

$$\boldsymbol{x} = T(\boldsymbol{q}), \qquad T(0) = 0 \tag{7}$$

which gives a new representation of the system equations (5) and (6) of the form[22]

$$\dot{\boldsymbol{x}} = A_0\boldsymbol{x} + \phi(y, \beta), \qquad y = [1, 0, 0, 0]\boldsymbol{x} \tag{8}$$

where $\phi \in R^4$ is a vector function of the output and input variables, $U_0$ is an open set in $R^4$ containing the origin, and

$$A_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The system equation (8) is said to be in an output feedback form.

Define the Lie derivative of $c(\boldsymbol{q})$ along the vector field $\boldsymbol{f}$ as $L_f c(\boldsymbol{q}) = [\partial c(\boldsymbol{q})/\partial \boldsymbol{q}]\boldsymbol{f}(\boldsymbol{q})$, the Lie bracket of any two vector fields $\boldsymbol{v}(\boldsymbol{q})$ and $\boldsymbol{w}(\boldsymbol{q})$ as $[\boldsymbol{v}, \boldsymbol{w}] = (\partial \boldsymbol{w}/\partial \boldsymbol{q})\boldsymbol{v} - (\partial \boldsymbol{v}/\partial \boldsymbol{q})\boldsymbol{w}$, and $dc(\boldsymbol{q}) = \partial c(\boldsymbol{q})/\partial \boldsymbol{q}$. The Lie bracket of $[\boldsymbol{v}, \boldsymbol{w}]$ is also denoted as $ad_v^i \boldsymbol{w} = ad_v(ad_v^{i-1}\boldsymbol{w})$, $ad_v^1 \boldsymbol{w} = ad_v \boldsymbol{w}$, and $ad_v^0 \boldsymbol{w} = \boldsymbol{w}$. Let $L_f^i c = L_f L_f^{i-1} c$ and $L_f^0 c = c$. The following lemma provides a necessary and sufficient condition for the existence of an output feedback form of the system equations (5) and (6).[22]

*Lemma*: There exists a local diffeomorphism in an open set $U_0 \in R^4$ containing the origin $\boldsymbol{q} = 0, \boldsymbol{x} = T(\boldsymbol{q})$ with $T(0) = 0, \boldsymbol{x} \in R^4$ that transforms the system equations (5) and (6) into an output feedback form Eq. (8) if and only if, in $U_0$, the following conditions hold: 1) rank $\{dc, d(L_f c), \ldots, d(L_f^3 c)\} = 4$, the dimension of the state space, 2) $[ad_f^i \boldsymbol{r}, ad_f^j \boldsymbol{r}] = 0, 0 \le i, j \le 3$, and 3) $[\bar{\boldsymbol{g}}, ad_f^i \boldsymbol{r}] = 0, 0 \le j \le 2$ with $\boldsymbol{r} \in R^4$ being the vector field solution of

$$\begin{bmatrix} \langle dc, \boldsymbol{r} \rangle \\ \langle dL_f c, \boldsymbol{r} \rangle \\ \langle dL_f^2 c, \boldsymbol{r} \rangle \\ \langle dL_f^3 c, \boldsymbol{r} \rangle \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{9}$$

where $\langle d(L_f^i c), \boldsymbol{r} \rangle = (\partial L_f^i c / \partial \boldsymbol{q}) \boldsymbol{r}$. Also $T$ is a global diffeomorphism if and only if conditions 1–3 hold in $R^4$ and, in addition, 4) $ad_f^i \boldsymbol{r}, 0 \le i \le 3$, are complete vector fields.

For the existence of an output feedback form of the aeroelastic model, the following result is stated.

*Corollary*: Consider the aeroelastic system equations (5) and (6) for which the output is $y = c(\boldsymbol{q}) = h$, the plunge displacement. Then there does not exist a local diffeomorphism transforming the system into an output feedback form. However, for the output $y = c(\boldsymbol{q}) = \alpha$, the pitch angle, a global diffeomorphism exists that gives a new representation equation (8) of the aeroelastic model.

*Proof*: The proof of the negative result for $y = h$ is provided in the Appendix. For the choice of the output $y = \alpha$, it can be shown that the lemma holds for all $\boldsymbol{q} \in R^4$, and, therefore, an output feedback form exists.

Apparently, because of nonexistence of an output feedback form, plunge displacement trajectory control by feedback of $h$ based on the backstepping technique[21] is not feasible. However, using the adaptive control system of Ref. 16, one can accomplish plunge displacement control by feedback of $h$ and $\alpha$.

### B. Output Feedback Form: Pitch Angle as Output

In this subsection, a representation of the system in an output form is derived for which the pitch angle is the output. According to the corollary, for $y = \alpha$, an output feedback form exists. Now a linear transformation $\boldsymbol{x} = T\boldsymbol{q}$, where $T$ is a $4 \times 4$ nonsingular matrix, is obtained for transforming Eqs. (5) and (6) into Eq. (8).

Let $y = c(\boldsymbol{q}) = \alpha$, the pitch angle, be the measured output. For the derivation of an output feedback form, it will be convenient to write Eq. (5) as

$$\dot{\boldsymbol{q}} = Q\boldsymbol{q} + \boldsymbol{v}$$

where

$$\boldsymbol{v}(t) = \bar{\boldsymbol{p}} k_{n\alpha}(\alpha) + \bar{\boldsymbol{g}} \beta \tag{10}$$

is treated as an input vector and one sets $\boldsymbol{f} = Q\boldsymbol{q}$ to applying the lemma. Then when $\boldsymbol{f} = Q\boldsymbol{q}$ is used, it is easy to check that

$$dc(\boldsymbol{q}) = \frac{\partial \alpha}{\partial \boldsymbol{q}} = [1, 0, 0, 0]\boldsymbol{q} \triangleq C\boldsymbol{q} \tag{11}$$

$$L_f^j C(\boldsymbol{q}) = CQ^j \boldsymbol{q} \tag{12}$$

$$d\left[L_f^j C(\boldsymbol{q})\right] = CQ^j, \qquad j \ge 0 \tag{13}$$

The rank condition 1 of the lemma can be satisfied if and only if

$$O_e = [C^T, (CQ)^T, \ldots, (CQ^3)^T]^T \tag{14}$$

is nonsingular. Note that $O_e$ is the observability matrix of the linearized model Eq. (1). For model (1), $O_e$ is found to be nonsingular. Because $\langle dL_f^j C, \boldsymbol{r} \rangle = CQ^j \boldsymbol{r}$, Eq. (9) yields

$$\begin{bmatrix} C\boldsymbol{r} \\ CQ\boldsymbol{r} \\ CQ^2\boldsymbol{r} \\ CQ^3\boldsymbol{r} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{15}$$

In view of the nonsingularity of $O_e$ for the observable aeroelastic system, Eq. (15) has a unique solution for $r$

$$\boldsymbol{r} = O_e^{-1} [0, 0, 0, 1]^T$$

Then a global change of coordinates $\boldsymbol{x} = T\boldsymbol{q}$ (Ref. 22) transforms Eq. (3) into an output feedback form, where

$$T^{-1} = [Q^3\boldsymbol{r}, Q^2\boldsymbol{r}, Q\boldsymbol{r}, \boldsymbol{r}] \tag{16}$$

Let

$$\det(\lambda I - Q) = \lambda^4 + n_3\lambda^3 + n_2\lambda^2 + n_1\lambda + n_0 \tag{17}$$

By the Caley–Hamilton theorem one has

$$Q^4 = -n_3 Q^3 - n_2 Q^2 - n_1 Q - n_0 I \tag{18}$$

In view of Eq. (18), when $TQT^{-1}$ is evaluated, it can be easily seen that an output feedback form of Eq. (3) is similar to Eq. (8) and is given by[22]

$$\dot{x} = \begin{bmatrix} -\bar{\boldsymbol{n}} & I_{3\times 3} \\ -n_0 & 0_{1\times 3} \end{bmatrix} \boldsymbol{x} + \boldsymbol{p} k_{n_\alpha}(\alpha) + \boldsymbol{g}\beta \tag{19}$$

$$\dot{\boldsymbol{x}} \triangleq A_0 \boldsymbol{x} + \phi(\alpha, \beta) \tag{20}$$

where $\phi(\alpha, \beta) = -\boldsymbol{n} x_1 + \boldsymbol{p} k_{n\alpha}(\alpha) + \boldsymbol{g}\beta$, $\bar{\boldsymbol{n}} = [n_3, n_2, n_1]^T$, $\boldsymbol{n} = [\bar{\boldsymbol{n}}^T, n_0]^T$, $\boldsymbol{g} = T\bar{\boldsymbol{g}} = [g_1, g_2, g_3, g_4]^T$, $\boldsymbol{p} = T\bar{\boldsymbol{p}} = [p_1, p_2, p_3, p_4]^T$, $\boldsymbol{x} = [x_1, x_2, x_3, x_4]^T$, and $y = x_1 = \alpha$. Then in view of Eq. (4), one has

$$\dot{x}_1 = \dot{y} = C\dot{\boldsymbol{q}} = C\left[Q\boldsymbol{q} + \bar{\boldsymbol{p}} k_{n_\alpha} + \bar{\boldsymbol{g}}\beta\right] = CQ\boldsymbol{q} = CQT^{-1}\boldsymbol{x} \tag{21}$$

which, using Eqs. (15), (16), and (18), yields

$$\dot{x}_1 = C[Q^4\boldsymbol{r}, Q^3\boldsymbol{r}, Q^2\boldsymbol{r}, Q\boldsymbol{r}]\boldsymbol{x} = [-n_3, 1, 0, 0]\boldsymbol{x} \tag{22}$$

This implies that that $p_1 = g_1 = 0$. Define

$$\phi(\alpha, \beta) = -\boldsymbol{n}\alpha + \boldsymbol{p} k_{n_\alpha} + \boldsymbol{g}\beta \doteq F^T(\alpha, \beta)\boldsymbol{\theta} \tag{23}$$

where $\boldsymbol{\theta} = (g_2, g_3, g_4, -\boldsymbol{n}^T, p_2 \boldsymbol{p}_\alpha, p_3 \boldsymbol{p}_\alpha, p_4 \boldsymbol{p}_\alpha,)^T \in R^{19}$, $\boldsymbol{p}_\alpha = (k_{\alpha_1}, k_{\alpha_2}, k_{\alpha_3}, k_{\alpha_4})$, and the $4 \times 19$ matrix $F^T$ is

$$F^T = \big(\beta\boldsymbol{e}_2, \beta\boldsymbol{e}_3, \beta\boldsymbol{e}_4, \alpha\boldsymbol{e}_1, \alpha\boldsymbol{e}_2, \alpha\boldsymbol{e}_3, \alpha\boldsymbol{e}_4, \alpha^2\boldsymbol{e}_2, \alpha^3\boldsymbol{e}_2, \alpha^4\boldsymbol{e}_2, \alpha^5\boldsymbol{e}_2,$$
$$\alpha^2\boldsymbol{e}_3, \alpha^3\boldsymbol{e}_3, \alpha^4\boldsymbol{e}_3, \alpha^5\boldsymbol{e}_3, \alpha^2\boldsymbol{e}_4, \alpha^3\boldsymbol{e}_4, \alpha^4\boldsymbol{e}_4, \alpha^5\boldsymbol{e}_4\big)$$

Here $\boldsymbol{e}_k$ is a vector of appropriate dimension whose $k$th element is one and the remaining elements are zero.

### C. Filters and State Estimation

In the absence of full state measurement, it is necessary to design certain filters for obtaining an estimate of the state vector. When the definition of matrix $F$ is used, Eq. (19) can be written as

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + \boldsymbol{l}\alpha + F^T(\alpha, \beta)\boldsymbol{\theta} \tag{24}$$

where $\boldsymbol{l} = (l_1, l_2, l_3, l_4)^T = (\bar{\boldsymbol{l}}^T, l_4)^T$ and

$$A = \begin{bmatrix} -\bar{\boldsymbol{l}} & I_{3\times 3} \\ -l_4 & 0_{1\times 3} \end{bmatrix}$$

The vector $\boldsymbol{l}$ is chosen so that $A$ is a stable matrix. Equation (24) is a canonical representation of the system equation (1) in which $A$ is in a special from, the regressor matrix $F$ is a function of the measured variable $\alpha$ and the input $\beta$, and all unknown parameters of the system are included in the vector $\boldsymbol{\theta}$.

Now based on Ref. 21, certain filters are designed for estimating the state vector $\boldsymbol{x}$. In view of Eq. (24), following Ref. 21, filters are given by

$$\dot{\boldsymbol{\xi}} = A\boldsymbol{\xi} + \boldsymbol{l}\alpha, \qquad \dot{\Omega}^T = A\Omega^T + F^T(\alpha, \beta) \tag{25}$$

where $\boldsymbol{\xi} \in R^4$ and $\Omega^T \in R^{4\times 19}$. Define a state estimate as

$$\hat{\boldsymbol{x}} = \boldsymbol{\xi} + \Omega^T\boldsymbol{\theta} \tag{26}$$

and let the state error be $\tilde{\boldsymbol{x}} = (\boldsymbol{x} - \hat{\boldsymbol{x}})$. When Eqs. (24–26) are used, it follows that the error $\tilde{\boldsymbol{x}}$ is governed by

$$\dot{\tilde{\boldsymbol{x}}} = A\tilde{\boldsymbol{x}} \tag{27}$$

Because $A$ is a Hurwitz matrix, $\tilde{\boldsymbol{x}}(t) \to 0$ as $t \to \infty$ and, therefore, $\hat{\boldsymbol{x}}(t)$ asymptotically converges to $\boldsymbol{x}(t)$.

For simplicity in synthesis, and in view of the special form of the matrix $F$, one can reduce the dimension of the $\boldsymbol{\Omega}$ filter. Define

$$\Omega^T = [\boldsymbol{v}_2, \boldsymbol{v}_1, \boldsymbol{v}_0, \boldsymbol{s}_1, \boldsymbol{s}_2, \ldots, \boldsymbol{s}_{16}] \doteq [\boldsymbol{v}_2, \boldsymbol{v}_1, \boldsymbol{v}_0, S] \tag{28}$$

where each column of $\Omega^T$ is a $4 \times 1$ vector. Because of the special structure of $F^T$, it follows from Eq. (25) that $\boldsymbol{v}_i$ and $\boldsymbol{s}_i$ satisfy

$$\dot{\boldsymbol{v}}_2 = A\boldsymbol{v}_2 + \boldsymbol{e}_2\beta, \qquad \dot{\boldsymbol{v}}_1 = A\boldsymbol{v}_1 + \boldsymbol{e}_3\beta, \qquad \dot{\boldsymbol{v}}_0 = A\boldsymbol{v}_0 + \boldsymbol{e}_4\beta$$

$$\dot{\boldsymbol{s}}_1 = A\boldsymbol{s}_1 + \boldsymbol{e}_1\alpha, \qquad \dot{\boldsymbol{s}}_2 = A\boldsymbol{s}_2 + \boldsymbol{e}_2\alpha, \qquad \dot{\boldsymbol{s}}_3 = A\boldsymbol{s}_3 + \boldsymbol{e}_3\alpha$$

$$\dot{\boldsymbol{s}}_4 = A\boldsymbol{s}_4 + \boldsymbol{e}_4\alpha, \qquad \dot{\boldsymbol{s}}_5 = A\boldsymbol{s}_5 + \boldsymbol{e}_2\alpha^2, \qquad \dot{\boldsymbol{s}}_6 = A\boldsymbol{s}_6 + \boldsymbol{e}_2\alpha^3$$

$$\dot{\boldsymbol{s}}_7 = A\boldsymbol{s}_7 + \boldsymbol{e}_2\alpha^4, \qquad \dot{\boldsymbol{s}}_8 = A\boldsymbol{s}_8 + \boldsymbol{e}_2\alpha^5, \qquad \dot{\boldsymbol{s}}_9 = A\boldsymbol{s}_9 + \boldsymbol{e}_3\alpha^2$$

$$\dot{\boldsymbol{s}}_{10} = A\boldsymbol{s}_{10} + \boldsymbol{e}_3\alpha^3, \qquad \dot{\boldsymbol{s}}_{11} = A\boldsymbol{s}_{11} + \boldsymbol{e}_3\alpha^4, \qquad \dot{\boldsymbol{s}}_{12} = A\boldsymbol{s}_{12} + \boldsymbol{e}_3\alpha^5$$

$$\dot{\boldsymbol{s}}_{13} = A\boldsymbol{s}_{13} + \boldsymbol{e}_4\alpha^2, \qquad \dot{\boldsymbol{s}}_{14} = A\boldsymbol{s}_{14} + \boldsymbol{e}_4\alpha^3, \qquad \dot{\boldsymbol{s}}_{15} = A\boldsymbol{s}_{14} + \boldsymbol{e}_4\alpha^4$$

$$\dot{\boldsymbol{s}}_{16} = A\boldsymbol{s}_{16} + \boldsymbol{e}_4\alpha^5 \tag{29}$$

When it is noted that $\boldsymbol{e}_3 = A\boldsymbol{e}_4$, $\boldsymbol{e}_2 = A\boldsymbol{e}_3$, and $\boldsymbol{e}_1 = A\boldsymbol{e}_2$, in view of Eq. (29) one finds that

$$\boldsymbol{v}_1 = A\boldsymbol{v}_0, \qquad \boldsymbol{v}_2 = A^2\boldsymbol{v}_0, \qquad \boldsymbol{s}_1 = A^3\boldsymbol{s}_4, \qquad \boldsymbol{s}_2 = A^2\boldsymbol{s}_4$$

$$\boldsymbol{s}_3 = A\boldsymbol{s}_4, \qquad \boldsymbol{s}_5 = A^2\boldsymbol{s}_{13}, \qquad \boldsymbol{s}_6 = A^2\boldsymbol{s}_{14}, \qquad \boldsymbol{s}_7 = A^2\boldsymbol{s}_{15}$$

$$\boldsymbol{s}_8 = A^2\boldsymbol{s}_{16}, \qquad \boldsymbol{s}_9 = A\boldsymbol{s}_{13}, \qquad \boldsymbol{s}_{10} = A\boldsymbol{s}_{14}, \qquad \boldsymbol{s}_{11} = A\boldsymbol{s}_{15}$$

$$\boldsymbol{s}_{12} = A\boldsymbol{s}_{16} \tag{30}$$

That is, $\boldsymbol{\Omega}$ can be obtained by using Eq. (30) for synthesis if $\boldsymbol{v}_0$ and $\boldsymbol{s}_k$, $k = 4$, 13, 14, 15, and 16, are available. Thus, for synthesis, one needs filters of dimension 24. Note that the state estimate $\hat{\boldsymbol{x}}$ cannot be computed using Eq. (26), but it is useful for the subsequent development.

## IV. Adaptive Pitch Angle Control Law

This section considers the design of control system using only pitch angle measurement. Let $y_r$ be a smooth trajectory that is to be tracked by $\alpha$. In view of Eqs. (24) and (26), the derivative of the controlled output variable $\alpha$ is given by

$$\dot{\alpha} = x_2 - n_3\alpha = \xi_2 + \Omega^T_{(2)}\boldsymbol{\theta} + \tilde{x}_2 - n_3\alpha \tag{31}$$

where $\Omega^T_{(k)}$, $\xi_k$, and $\tilde{x}_k$ denote $k$th rows of $\Omega^T$, $\boldsymbol{\xi}$, and $\tilde{\boldsymbol{x}}$, respectively. When the definitions of $\boldsymbol{\theta}$ and $\Omega^T$ are used, and when it is noted that $-n_3 = \boldsymbol{e}_4^T\boldsymbol{\theta}$, Eq. (31) yields

$$\dot{\alpha} = g_2 v_{22} + \xi_2 + \tilde{x}_2 + \bar{\boldsymbol{\omega}}^T\boldsymbol{\theta} \tag{32}$$

where $\bar{\boldsymbol{\omega}}^T = (0, v_{12}, v_{02}, S_{(2)}) + \boldsymbol{e}_4^T\alpha$ and $v_{ik}$ and $S_{(k)}$ are the $k$th rows of $\boldsymbol{v}_i$ and $S$, respectively. Because we are interested in the trajectory control of $y = \alpha$, consider the tracking error $z_1$ defined as

$$z_1 = y - y_r \tag{33}$$

Now the controller design is performed in two steps following a backstepping technique.[21]

Step 1 is as follows: The derivative of $z_1$ is

$$\dot{z}_1 = g_2 v_{22} + \xi_2 + \tilde{x}_2 + \bar{\boldsymbol{\omega}}^T\boldsymbol{\theta} - \dot{y}_r \tag{34}$$

Because $v_{22}$ is treated as a virtual control for controlling $z_1$, define

$$z_2 = v_{22} - \hat{\rho}\dot{y}_r - \alpha_1 \tag{35}$$

where $\hat{\rho}$ is an estimate of $\rho = g_2^{-1}$ and $\alpha_1$ is the stabilizing function yet to be chosen. Using Eq. (35) in Eq. (34) yields

$$\dot{z}_1 = \tilde{x}_2 + \xi_2 + g_2[z_2 + \alpha_1 + \hat{\rho}\dot{y}_r] + \bar{\boldsymbol{\omega}}^T\boldsymbol{\theta} - \dot{y}_r \tag{36}$$

The stabilizing function $\alpha_1$ is chosen as

$$\alpha_1 = \hat{\rho}\bar{\alpha}_1, \qquad \bar{\alpha}_1 = -c_1 z_1 - \xi_2 - \bar{\boldsymbol{\omega}}^T\hat{\boldsymbol{\theta}} - d_1 z_1 \tag{37}$$

where $c_i, d_i > 0$ and $\hat{\boldsymbol{\theta}}$ is an estimate of $\boldsymbol{\theta}$.

Note that $g_2\hat{\rho} = g_2(\rho - \tilde{\rho}) = 1 - g_2\tilde{\rho}$; it follows from Eqs. (36) and (37) that

$$\dot{z}_1 = g_2 z_2 - g_2\tilde{\rho}\bar{\alpha}_1 + \bar{\boldsymbol{\omega}}^T\tilde{\boldsymbol{\theta}} - d_1 z_1 + \tilde{x}_2 - g_2\tilde{\rho}\dot{y}_r - c_1 z_1 \tag{38}$$

Now consider a Lyapunov function of the form

$$V_1 = d_1^{-1}\tilde{\boldsymbol{x}}^T P\tilde{\boldsymbol{x}} + \left(z_1^2 + |g_2|\gamma^{-1}\tilde{\rho}^2\right)\big/2 \tag{39}$$

where $\gamma > 0$ and the positive definite symmetric matrix $P$ satisfies the Lyapunov equation

$$PA + A^T P = -I_{4\times 4} \tag{40}$$

Because $A$ is a stable matrix, $P$ is the unique solution of Eq. (40). The derivative of $V_1$ is given by

$$\dot{V}_1 = d_1^{-1}(\dot{\tilde{\boldsymbol{x}}}^T P\tilde{\boldsymbol{x}} + \tilde{\boldsymbol{x}}^T P\dot{\tilde{\boldsymbol{x}}}) + z_1\dot{z}_1 - |g_2|\gamma^{-1}\tilde{\rho}\dot{\hat{\rho}} \tag{41}$$

Substituting Eqs. (27), (38), and (40) in Eq. (41) yields

$$\dot{V}_1 = g_2 z_1 z_2 + \bar{\boldsymbol{\omega}}^T\tilde{\boldsymbol{\theta}}z_1 - c_1 z_1^2 - \left(|\tilde{\boldsymbol{x}}|^2\big/d_1\right) - g_2\tilde{\rho}\bar{\alpha}_1 z_1 - d_1 z_1^2$$
$$+ \tilde{x}_2 z_1 - (\dot{\hat{\rho}}|g_2|/\gamma)\tilde{\rho} - g_2\tilde{\rho}\dot{y}_r z_1 \tag{42}$$

where $|\cdot|$ denotes the Euclidean norm of a vector. Using Young's inequality,[21] one has

$$\tilde{x}_2 z_1 \le |\tilde{x}_2||z_1| \le d_1 z_1^2 + \tilde{x}_2^2\big/(4d_1) \le d_1 z_1^2 + |\tilde{\boldsymbol{x}}|^2/(4d_1) \tag{43}$$

Using Eq. (43) in (42) yields

$$\dot{V}_1 \le g_2 z_1 z_2 + \bar{\boldsymbol{\omega}}^T\tilde{\boldsymbol{\theta}}z_1 - c_1 z_1^2 - \left(3|\tilde{\boldsymbol{x}}|^2\big/4d_1\right)$$
$$+ z_1\tilde{\rho}[-g_2\bar{\alpha}_1 - g_2\dot{y}_r] - (\dot{\hat{\rho}}|g_2|\tilde{\rho}/\gamma) \tag{44}$$

Because $\tilde{\rho}$ is unknown, this can be eliminated from Eq. (44) by choosing an update law of the form

$$\dot{\hat{\rho}} = -\gamma\,\mathrm{sign}(g_2)[z_1(\bar{\alpha}_1 + \dot{y}_r)] \tag{45}$$

Now substituting the update law in Eq. (44) yields

$$\dot{V}_1 \le -c_1 z_1^2 - (3/4d_1)|\tilde{\boldsymbol{x}}|^2 + \bar{\boldsymbol{\omega}}^T\tilde{\boldsymbol{\theta}}z_1 + g_2 z_1 z_2 \tag{46}$$

Step 2 is as follows: The derivative of $z_2$ is given by

$$\dot{z}_2 = \dot{v}_{22} - \dot{\hat{\rho}}\dot{y}_r - \hat{\rho}\ddot{y}_r - \dot{\alpha}_1 \tag{47}$$

Because $\alpha_1$ is a function of $\hat{\rho}$, $\xi_2$, $S_{(2)}$, $v_{12}$, $v_{02}$, $y_r$, $\hat{\theta}$, and $x_1$, its derivative is given by

$$\dot{\alpha}_1 = \frac{\partial\alpha_1}{\partial v_{12}}\dot{v}_{12} + \frac{\partial\alpha_1}{\partial v_{02}}\dot{v}_{02} + \frac{\partial\alpha_1}{\partial\hat{\rho}}\dot{\hat{\rho}} + \frac{\partial\alpha_1}{\partial\xi_2}\dot{\xi}_2 + \frac{\partial\alpha_1}{\partial S_{(2)}}\dot{S}^T_{(2)}$$
$$+ \frac{\partial\alpha_1}{\partial y_r}\dot{y}_r + \frac{\partial\alpha_1}{\partial\hat{\theta}}\dot{\hat{\theta}} + \frac{\partial\alpha_1}{\partial x_1}\dot{x}_1 \doteq a_0 + \left(\frac{\partial\alpha_1}{\partial x_1}\right)\dot{x}_1 \tag{48}$$

where $a_0$ is obtained by comparing terms in Eq. (48). For the computation of $a_0$, the derivatives of various signals are substituted in Eq. (48), but $\dot{\hat{\boldsymbol{\theta}}}$ is yet to be determined.

Adding and subtracting appropriate $\hat{\boldsymbol{\theta}}$-dependent terms and using Eq. (31) in (48) yields

$$\dot{\alpha}_1 = a_0 + \left(\frac{\partial \alpha_1}{\partial x_1}\right)\left(\xi_2 + \Omega_{(2)}^T \boldsymbol{\theta} + \tilde{x}_2 + \theta_4 \alpha\right) \doteq a_1 + a_2 \tilde{x}_2 + \boldsymbol{a}_3^T \tilde{\boldsymbol{\theta}} \tag{49}$$

where

$$a_1 = a_0 + \frac{\partial \alpha_1}{\partial x_1}\left(\xi_2 + \Omega_{(2)}^T \hat{\boldsymbol{\theta}} + \hat{\theta}_4 \alpha\right), \qquad a_2 = \frac{\partial \alpha_1}{\partial x_1}$$

$$\boldsymbol{a}_3^T = \frac{\partial \alpha_1}{\partial x_1}\left(\Omega_{(2)}^T + \boldsymbol{e}_4^T \alpha\right)$$

Substituting Eq. (49) in (47) yields

$$\dot{z}_2 = -l_2 v_{21} + v_{23} + \beta - \dot{\hat{\rho}} \dot{y}_r - \hat{\rho} \ddot{y}_r - a_1 - a_2 \tilde{x}_2 - \boldsymbol{a}_3^T \tilde{\boldsymbol{\theta}}$$

$$\doteq a^* - a_2 \tilde{x}_2 - \boldsymbol{a}_3^T \tilde{\boldsymbol{\theta}} + \beta \tag{50}$$

where $a^* = -l_2 v_{21} + v_{23} - \dot{\hat{\rho}} \dot{y}_r - \hat{\rho} \ddot{y}_r - a_1$.

In view of Eq. (50), we choose control $\beta$ as

$$\beta = -a^* - c_2 z_2 - d_2 |a_2|^2 z_2 - \hat{g}_2 z_1 \tag{51}$$

Substituting $\beta$ in Eq. (50) yields

$$\dot{z}_2 = -c_2 z_2 - d_2 |a_2|^2 z_2 - \hat{g}_2 z_1 - a_2 \tilde{x}_2 - \boldsymbol{a}_3^T \tilde{\boldsymbol{\theta}} \tag{52}$$

Now consider a Lyapunov function

$$V_2 = V_1 + d_2^{-1} \tilde{\boldsymbol{x}}^T P \tilde{\boldsymbol{x}} + \left(z_2^2 + \tilde{\boldsymbol{\theta}}^T \Gamma^{-1} \tilde{\boldsymbol{\theta}}\right)/2 \tag{53}$$

where $\Gamma$ is a positive definite symmetric matrix. In view of Eq. (40), the derivative of $V_2$ is given by

$$\dot{V}_2 = \dot{V}_1 - \left(|\tilde{\boldsymbol{x}}|^2/d_2\right) + z_2 \dot{z}_2 - \tilde{\boldsymbol{\theta}}^T \Gamma^{-1} \dot{\hat{\boldsymbol{\theta}}} \tag{54}$$

Using Eq. (46) in (54) and noting that $g_2 = \theta_1$ yields

$$\dot{V}_2 \le -c_1 z_1^2 - \left(3|\tilde{\boldsymbol{x}}|^2/4d_1\right) + \bar{\boldsymbol{\omega}}^T \tilde{\boldsymbol{\theta}} z_1 + (\hat{\theta}_1 + \tilde{\theta}_1) z_1 z_2 - c_2 z_2^2$$

$$- d_2 z_2^2 |a_2|^2 - \hat{\theta}_1 z_1 z_2 - z_2 a_2 \tilde{x}_2 - \boldsymbol{a}_3^T \tilde{\boldsymbol{\theta}} z_2$$

$$- \tilde{\boldsymbol{\theta}}^T \Gamma^{-1} \dot{\hat{\boldsymbol{\theta}}} - \left(|\tilde{\boldsymbol{x}}|^2/d_2\right) \tag{55}$$

Define

$$\boldsymbol{\tau} = (\bar{\boldsymbol{\omega}} z_1 + \boldsymbol{e}_1 z_1 z_2 - \boldsymbol{a}_3 z_2) \tag{56}$$

Using Young's inequality, one has

$$|z_2 a_2 \tilde{x}_2| \le |z_2||a_2||\tilde{x}_2| \le d_2 z_2^2 \left|a_2^2\right| + \left(\left|\tilde{x}_2^2\right|/4d_2\right) \tag{57}$$

Substituting Eqs. (56) and (57) in (55) and noting that $|\tilde{x}_2|^2 \le |\tilde{\boldsymbol{x}}|^2$, one has

$$\dot{V}_2 \le -c_1 z_1^2 - c_2 z_2^2 - \tfrac{3}{4}\left(d_1^{-1} + d_2^{-1}\right)|\tilde{\boldsymbol{x}}|^2 + \tilde{\boldsymbol{\theta}}^T \left(\boldsymbol{\tau} - \Gamma^{-1} \dot{\hat{\boldsymbol{\theta}}}\right) \tag{58}$$

Now one chooses the adaptation law for $\hat{\theta}$ as

$$\dot{\hat{\boldsymbol{\theta}}} = \Gamma \boldsymbol{\tau} \tag{59}$$

which yields

$$\dot{V}_2 \le -c_1 z_1^2 - c_2 z_2^2 - \tfrac{3}{4}\left(d_1^{-1} + d_2^{-1}\right)|\tilde{\boldsymbol{x}}|^2 \tag{60}$$

Because $V_2$ is a positive definite function and $\dot{V}_2 \le 0$, it follows that $z_i$, $\rho$, $\tilde{\boldsymbol{\theta}}$, and $\tilde{\boldsymbol{x}}$ are bounded. Now when arguments are used similar to those used in the stability proof of Ref. 21 (pp. 340–342) and it is assumed that the zero dynamics[23] are stable, the following result can be stated.

*Theorem*: Consider the closed-loop system Eqs. (3), (45), (51), and (59). Suppose that $y_r$ is a bounded, smooth reference trajectory converging to zero and that the zero dynamics of the system are stable. Then the solution of system equation (1) beginning from any initial condition $\boldsymbol{q}(0) \in R^4$ is such that the tracking error $(\alpha - y_r)$ tends to zero and the state $\boldsymbol{q}$ converges to the origin as $t \to \infty$.

The zero dynamics describe the internal dynamics of the system when the output $y = \alpha$ is identically zero. For the control of $\alpha$, the theorem assumes that the zero dynamics are stable. The transfer function of the linearized system Eq. (19), that is, $k_{n\alpha} = 0$, is

$$\hat{y}(s)/\hat{\beta}(s) = N(s) D^{-1}(s)$$

where $\hat{(\cdot)}$ denotes the Laplace transform, $N(s) = g_2 s^2 + g_3 s + g_4$, and $D(s) = s^4 + n_3 s^3 + n_2 s^2 + n_1 s + n_0$. The system has stable zero dynamics if the polynomial $N$ is Hurwitz. The stability properties of zero dynamics have been extensively examined in Refs. 13, 14, and 16. Note that stability of the zero dynamics is essential even in the nonadaptive output trajectory control systems.

## V. Simulation Results

In this section, numerical results for the pitch angle control are presented. The parameters of the system are given in the Appendix. Simulation is performed for different values of $a$ and $U$. A third-order filter of the form

$$(s^3 + 3\lambda s^2 + 3\lambda^2 s + \lambda^3) y_r = 0 \tag{61}$$

is used to obtain exponentially decaying trajectories to zero, where $\lambda > 0$. The initial conditions selected are $\alpha(0) = 5.75$ deg, $h(0) = 0.01$ m, $\dot{h}(0) = 0$, and $\dot{\alpha}(0) = 0$ deg/s. The initial conditions of the command generator are set as $y_r(0) = 5.75$ deg and $\dot{y}_r(0) = \ddot{y}_r(0) = 0$. The initial conditions for the parameters $\hat{g}_2(0)$, $\hat{g}_3(0)$, $\hat{g}_4(0)$, $\hat{\boldsymbol{\theta}}$, and $\hat{\rho}$ are assumed to be zero. Of course, this is rather a bad choice of parameter estimates, but this selection has been made to show the robustness of the controller. The initial states of the filters are set as $\boldsymbol{\Omega}(0) = 0$ and $\boldsymbol{\xi}(0) = 0$. The design parameters are selected as $c_1 = c_2 = d_1 = d_2 = 70$, $\gamma = 0.015$, $\Gamma = 300 I_{19 \times 19}$, $l_1 = 20$, $l_2 = 150$, $l_3 = 500$, and $l_4 = 625$. For these values of $l_i$, the eigenvalues of $A$ are at $-5$. Note that the origin of the open-loop model Eq. (1) is unstable. For the uncontrolled system, persistent periodic oscillations exist (Fig. 2).

### A. Case 1

The closed-loop system Eq. (1) with the control law Eq. (51) and the update laws Eqs. (45) and (59) for $a = -0.6847$ and $U = 16$ m/s is simulated. For the chosen value of $a$ and $U$, one has $g_2 = -0.0182$, $g_3 = -0.0629$, and $g_4 = -4.6190$ and the zeros of the transfer function [i.e., the roots of $N(s) = g_2 s^2 + g_3 s + g_4 = 0$] of the linearized model Eq. (1) are at $-1.7274 + 15.8340i$ and $-1.7274 - 15.8340i$ in the complex plane. Thus, the zero dynamics are stable. Selected responses are shown in Fig. 3. We observe that, after an initial

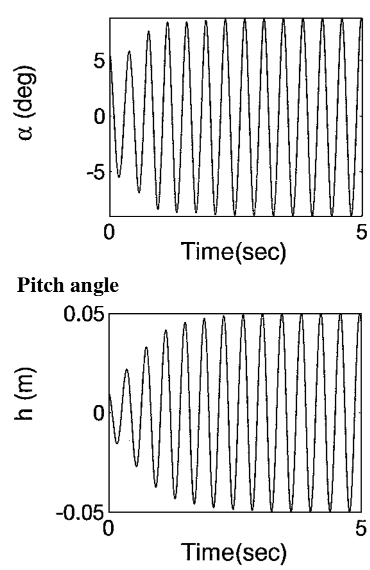
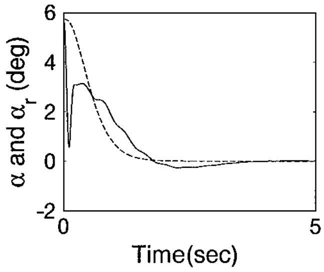
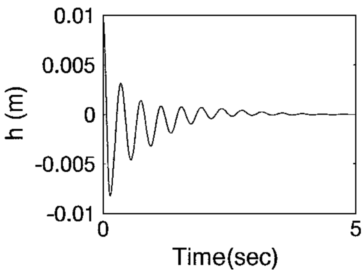


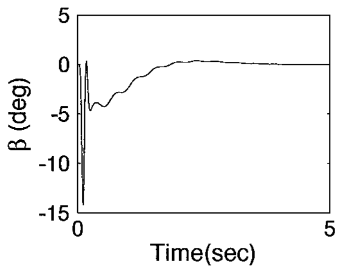

**Fig. 2 Open-loop response.**



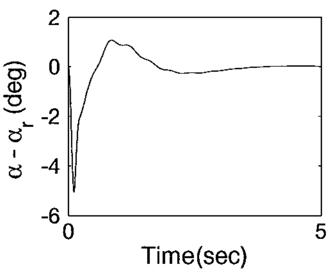

**Pitch angle $\alpha$ and $\alpha_r$**




**Plunge displacement $h$**




**Control input $\beta$**




**Error $\alpha - \alpha_r$**

**Fig. 3 Pitch angle control $a = -0.6847$ and $U = 16$ m/s.**

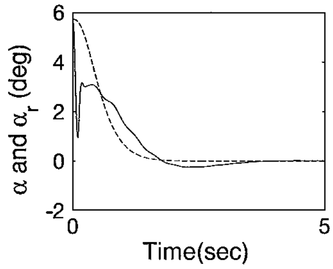
transient, the pitch angle asymptotically tracks the command trajectory. The response time is of the order of 4 s. Only a small control magnitude (less than 15 deg) is required for control. Because the zero dynamics are stable, the plunge displacement also converges to zero as predicted.

### B. Case 2

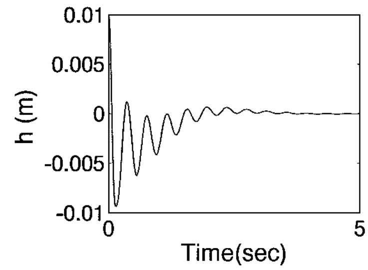
To examine the sensitivity of the controller with respect to the freestream velocity $U$, the closed-loop system with a different value of $U = 20$ m/s is simulated. For this new value of $U$, one has $g_2 = -0.0284$, $g_3 = -0.1055$, and $g_4 = -7.2171$ and the zeros of the transfer function are at $-1.8534 + 15.8197i$ and $-1.8534 - 15.8197i$, which are stable. We observe that the pitch angle asymptotically tracks the command trajectory (Fig. 4), but smaller control magnitude (less than 12 deg) is required due to increased effectiveness of the control surface. The response time is of the order of 4 s. The plunge displacement also converges to zero as predicted.

### C. Case 3

To examine the sensitivity of the controller with respect to parameter $a$, the closed-loop system for a different value of $a = -0.8$, but with $U = 16$ m/s as in case 1 is simulated. For the chosen values of $a$ and $U$, one has $g_2 = -0.0167$, $g_3 = -0.0547$, and $g_4 = -4.3960$ and the zeros are located at $-1.6393 + 16.1436i$ and $-1.6393 - 16.1436i$. We observe that, although the pitch angle asymptotically tracks the command trajectory (Fig. 5), slightly larger control magnitude (about 16 deg) is required. The response time is of the order of 4 s. The plunge displacement also converges to zero as predicted because the zero dynamics are stable.

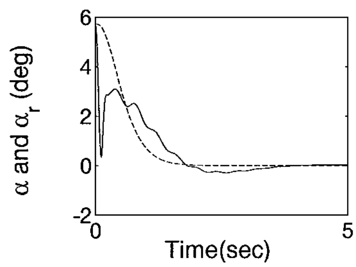



**Pitch angle $\alpha$ and $\alpha_r$**

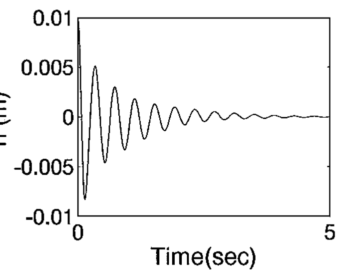



**Plunge displacement $h$**

**Fig. 4 Pitch angle control $a = -0.6847$ and $U = 20$ m/s.**




**Pitch angle $\alpha$ and $\alpha_r$**




**Plunge displacement $h$**

**Fig. 5 Pitch angle control $a = -0.8$ and $U = 16$ m/s.**

## VI. Conclusions

The existence of output feedback forms and the adaptive stabilization of an aeroelastic system using a single output measurement were considered. It was shown that, for the choice of the plunge displacement as an output, one cannot obtain an output feedback form and design an adaptive controller using backstepping technique with output feedback. Then an output feedback form of the model with the pitch angle as an output was obtained, and an adaptive stabilizer using only pitch angle feedback was designed. In the closed-loop system, asymptotic regulation of the state vector to the origin was accomplished. Simulation results were presented that showed that control of the pitch angle and the regulation of the state vector to the origin can be accomplished using only the pitch angle feedback in spite of the uncertainties in the system parameters. The adaptive controller has several design parameters that can be adjusted to obtain desirable response characteristics.

## Appendix: Proof of Corollary, System Parameters, and System Variables

The proof of the corollary follows. It is shown here that, for $y = c(\boldsymbol{q}) = h$, the plunge displacement, the lemma does not hold and that there does not exist an output feedback form. Computing the Lie derivatives gives

$$L_f^0 c = h \tag{A1}$$

$$L_f^1 c = \frac{\partial c}{\partial \boldsymbol{q}} \boldsymbol{f}(\boldsymbol{q}) = \dot{h} \tag{A2}$$

$$L_f^2 c = \frac{\partial \dot{h}}{\partial \boldsymbol{q}} \boldsymbol{f}(\boldsymbol{q}) = [0, 0, 0, 1]\boldsymbol{f}(\boldsymbol{q}) = M_{(2)}\boldsymbol{q} + p_{02}k_{n\alpha}(\alpha) \tag{A3}$$

$$L_f^3 c = \left(\frac{\partial L_f^2 c}{\partial \boldsymbol{q}}\right)\boldsymbol{f}(\boldsymbol{q})$$

$$= M_{(2)}Q\boldsymbol{q} + (m_{23}p_{01} + m_{24}p_{02})k_{n\alpha}(\alpha) + p_{02}k'_{n\alpha}(\alpha)\dot{\alpha} \tag{A4}$$

where $M_{(2)}$ is the second row of $M = [M_1, M_2]$ and $M_{ik}$ is the $ik$th element of $M$ and $k'_{n\alpha}(\alpha) = \partial k_{n\alpha}/\partial\alpha$. Using Eqs. (A1–A4), one has [note that $d(\cdot) = \partial(\cdot)/\partial\boldsymbol{q}$]

$$dc = [0, 1, 0, 0], \qquad d(L_f c) = [0, 0, 0, 1]$$

$$d\left(L_f^2 c\right) = \left[M_{(2)} + p_{02}k'_{n\alpha}\boldsymbol{e}_1^T\right]$$

$$d(Lf^3 c) = \Big[M_{(2)}Q + (m_{23}p_{01} + m_{24}p_{02})k'_{n\alpha}\boldsymbol{e}_1^T$$

$$+ p_0 2k''_{n\alpha}\dot{\alpha}\boldsymbol{e}_1^T + p_{02}k'_{n\alpha}\boldsymbol{e}_3^T\Big]$$

where $\boldsymbol{e}_i^T \in R^4$, a row vector in which the $i$th element is 1 and the remaining elements are zero.

Let $\boldsymbol{r} = [r_1, r_2, r_3, r_4]^T$, then Eq. (9) of the lemma gives

$$\langle dc, \boldsymbol{r}\rangle = 0 \Rightarrow r_2 = 0, \qquad \langle d(L_f C, \boldsymbol{r}\rangle = 0 \Rightarrow r_4 = 0$$

and $r_1$ and $r_3$ can be obtained by solving the simplified form of Eq. (9),

$$\begin{bmatrix} \left\langle d\left(L_f^2 c, \boldsymbol{r}\right\rangle \\ \left\langle d\left(L_f^3 c\right), \boldsymbol{r}\right\rangle \end{bmatrix} = \begin{bmatrix} m_{21} + p_{02}k'_{n\alpha} & m_{23} \\ s(\alpha, \dot{\alpha}) & p_{02}k'_{n\alpha} \end{bmatrix} \begin{bmatrix} r_1 \\ r_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{A5}$$

where $s(\alpha, \dot{\alpha}) = M_{(2)}Q_1 + (m_{23}p_{01} + m_{24}p_{02})k'_{n\alpha} + p_{02}k''_{n\alpha}\dot{\alpha}$. Solving Eq. (A5) yields

$$\begin{bmatrix} r_1 \\ r_3 \end{bmatrix} = \frac{1}{\triangle(\alpha, \dot{\alpha})} \begin{bmatrix} -m_{23} \\ m_{21} + p_{02}k'_{n\alpha}(\alpha) \end{bmatrix} \tag{A6}$$

where $\triangle = (m_{21} + b_{02}k'_{n\alpha})p_{02}k'_{n\alpha} - m_{23}s(\alpha, \dot{\alpha})$. Thus, $r_1$ and $r_3$ are nonlinear functions of $\alpha$ and $\dot{\alpha}$. Condition 3 of the lemma for $j = 0$ given by

$$[\bar{\boldsymbol{g}}, \boldsymbol{r}] = \frac{\partial \boldsymbol{r}}{\partial \boldsymbol{q}}\bar{g} = \frac{\partial \boldsymbol{r}}{\partial \dot{\alpha}}g_{01} + \frac{\partial \boldsymbol{r}}{\partial \dot{h}}g_{02} = \frac{\partial \boldsymbol{r}}{\partial \dot{\alpha}}g_{01} = 0 \tag{A7}$$

implies that (note that $r_2 = 0$ and $r_4 = 0$)

$$\frac{\partial r_i}{\partial \dot{\alpha}} = 0, \qquad i = 1, 3 \tag{A8}$$

In of view of Eq. (A8), $r_1$ and $r_3$ must be independent of $\dot{\alpha}$, which is a contradiction according to Eq. (A6) in which they are functions of $\dot{\alpha}$. Thus, the lemma does not hold for $y = h$, and there does not exist an output feedback form in this case.

The system parameters are as follows:

$$b = 0.135\ \text{m}, \qquad k_h = 2844.4\ \text{N/m}, \qquad c_h = 27.43\ \text{Ns/m}$$

$$c_\alpha = 0.036\ \text{Ns}, \qquad \rho = 1.225\ \text{kg/m}^3, \qquad c_{l\alpha} = 6.28$$

$$c_{l\beta} = 3.358, \qquad c_{m\alpha} = (0.5 + a)c_{l\alpha}, \qquad c_{m\beta} = -0.635$$

$$m_T = 12.387\ \text{kg}, \qquad m_W = 2.0490\ \text{kg}$$

$$I_\alpha = m_W x_\alpha^2 b_s^2 + 0.0517\ \text{kgm}^2$$

$$x_\alpha = [0.0873 - (b_s + ab_s)]/b_s$$

The system variables are given by

$$d = m_T I_\alpha - m_W x_\alpha^2 b^2)$$

$$k_1 = I_\alpha k_h/d$$

$$k_2 = \left(I_\alpha \rho b c_{l\alpha} + m_W x_\alpha b^3 \rho c_{m\alpha}\right)\big/d$$

$$k_3 = -m_W x_\alpha b k_h/d$$

$$k_4 = \left(-m_W x_\alpha b^2 \rho c_{l\alpha} - m_T \rho b^2 c_{m\alpha}\right)\big/d$$

$$c_{11} = \left[I_\alpha(c_h + \rho U b c_{l\alpha}) + m_W x_\alpha \rho U b^3 c_{m\alpha}\right]\big/d$$

$$c_{21} = \Big[I_\alpha \rho U b^2 c_{l\alpha}\left(\tfrac{1}{2} - a\right) - m_W x_\alpha b c_\alpha$$

$$+ m_W x_\alpha \rho U b^4 c_{m\alpha}\left(\tfrac{1}{2} - a\right)\Big]\big/d$$

$$c_{31} = \left(-m_W x_\alpha b c_h - m_W x_\alpha \rho U b^2 c_{l\alpha} - m_T \rho U b^2 c_{m\alpha}\right)\big/d$$

$$c_{41} = \left[m_T c_\alpha - m_W x_\alpha \rho U b^3 c_{l\alpha}\left(\tfrac{1}{2} - a\right) - m_T \rho U b^3 c_{m\alpha}\left(\tfrac{1}{2} - a\right)\right]\big/d$$

$$g_{01} = U^2\left(m_W x_\alpha b^2 \rho c_{l\beta} + m_T \rho b^2 c_{m\beta}\right)\big/d$$

$$g_{02} = U^2\left(-I_\alpha \rho b c_{l\beta} - m_W x_\alpha b^3 \rho c_{m\beta}\right)\big/d$$

$$p_0 = \begin{bmatrix} -m_T/d \\ m_W x_a b/d \end{bmatrix}$$

## References


[1]Fung, Y. C., *An Introduction to the Theory of Aeroelasticity*, Wiley, New York, 1955, pp. 207–215.

[2]Dowell, E. H. (ed.), *A Modern Course in Aeroelasticity*, Kluwer Academic, Norwell, MA, 1995, Chap. 1.

[3]Zhao, L. C., and Yang, Z. C., "Chaotic Motions of an Airfoil with Nonlinear Stiffness in Incompressible Flow," *Journal of Sound and Vibration*, Vol. 128, No. 2, 1990, pp. 245–254.

[4]Friedmann, P. P., Guillot, D., and Presente, E., "Adaptive Control of Aeroelastic Instabilities in Transonic Flow and Its Scaling," *Journal of Guidance, Control, and Dynamics*, Vol. 20, No. 6, 1997, pp. 1190–1199.

[5]Waszak, M. R., "Robust Multivariable Flutter Suppression for Benchmark Active Control Technology Wind-Tunnel Model," *Journal of Guidance, Control, and Dynamics*, Vol. 24, No. 1, 2001, pp. 147–153.

[6]Scott, R. C., Hoadley, S. T., Wieseman, C. D., and Durham, M. H., "Benchmark Active Controls Technology Model Aerodynamic Data," *Journal of Guidance, Control, and Dynamics*, Vol. 23, No. 5, 2000, pp. 914–921.

[7]Bennett, R. M., Scott, R. C., and Wieseman, C. D., "Computational Test Cases for the Benchmark Active Controls Model," *Journal of Guidance, Control, and Dynamics*, Vol. 23, No. 5, 2000, pp. 922–929.

[8]Mukhopadhyay, V., "Transonic Flutter Suppression Control Law Design and Wind-Tunnel Test Results," *Journal of Guidance, Control, and Dynamics*, Vol. 23, No. 5, 2000, pp. 930–937.

[9]Kelkar, A. G., and Joshi, S. M., "Passivity-Based Robust Control with Application to Benchmark Controls Technology Wing," *Journal of Guidance, Control, and Dynamics*, Vol. 23, No. 5, 2000, pp. 938–947.

[10]Barker, J. M., and Balas, G. J., "Comparing Linear Parameter-Varying Gain-Scheduled Control Techniques for Active Flutter Suppression," *Journal of Guidance, Control, and Dynamics*, Vol. 23, No. 5, 2000, pp. 948–955.

[11]Guillot, D. M., and Friedmann, P. P., "Fundamental Aeroservoelastic Study Combining Unsteady Computational Fluid Mechanics with Adaptive Control," *Journal of Guidance, Control, and Dynamics*, Vol. 23, No. 6, 2000, pp. 1117–1126.

[12]Scott, R. C., and L. E. Pado., "Active Control of Wind-Tunnel Model Aeroelastic Response Using Neural Networks," *Journal of Guidance, Control, and Dynamics*, Vol. 23, No. 6, 2000, pp. 1100–1108.

[13]Ko, J., Kurdila, A. J., and Strganac, T. W., "Nonlinear Control of a Prototypical Wing Section with Torsional Nonlinearity," *Journal of Guidance, Control, and Dynamics*, Vol. 20, No. 6, 1997, pp. 1181–1189.

[14]Ko, J., and Strganac, T. W., "Stability and Control of a Structurally Nonlinear Aeroelastic System," *Journal of Guidance, Control, and Dynamics*, Vol. 21, No. 5, 1998, pp. 718–725.

[15]Block, J., and Strganac, T. M., "Applied Active Control for a Nonlinear Aeroelastic Structure," *Journal of Guidance, Control, and Dynamics*, Vol. 21, No. 6, 1998, pp. 838–845.

[16]Xing, W., and Singh, S. N., "Adaptive Output Feedback Control of a Nonlinear Aeroelastic Structure," *Journal of Guidance, Control, and Dynamics*, Vol. 23, No. 6, 2000, pp. 1109–1116.

[17]Strganac, T. W., Ko, J., Thompson, D. E., and Kurdila, A. J., "Identification and Control of Limit Cycle Oscillations in Aeroelastic Systems," *Journal of Guidance, Control, and Dynamics*, Vol. 23, No. 6, 2000, pp. 1127–1133.

[18]Zhang, R., and Singh, S. N., "Adaptive Output Feedback Control of an Aeroelastic System with Unstructured Uncertainties," *Journal of Guidance, Control, and Dynamics*, Vol. 24, No. 3, 2001, pp. 502–509.

[19]Prasanth, R. K., and Mehra, R. K., "Control of a Nonlinear Aeroelastic System Using Euler–Lagrange Theory," *Journal of Guidance, Control, and Dynamics*, Vol. 23, No. 6, 2000, pp. 1134–1139.

[20]Kanellakopoulos, I., Kokotovic, P. V., and Morse, A. S., "Systematic Design of Adaptive Controller for Feedback Linearizable Systems," *IEEE Transaction on Automatic Control*, Vol. 36, No. 11, 1991, pp. 1241–1253.

[21]Krstic, M., Kanellakopoulos, I., and Kokotovic, P. V., *Nonlinear Adaptive Control Design*, Wiley, New York, 1995, pp. 327–347.

[22]Marino, R., and Tomei, P., *Nonlinear Control Design*, Prentice–Hall, New York, 1995, pp. 203–213.

[23]Isidori, A., *Nonlinear Control Systems*, Springer-Verlag, New York, 1989, pp. 172–183.